from 8 → 16 are

from Ex 22-39 (a) determine the general expression for the difference quotient, and (b) use the difference quotient to compute the slope of the secant line connecting points at

$x = 1$ and $x = 3$

22) $y = f(x) = 4x^2 + 3$

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x+\Delta x) = 4(x+\Delta x)^2 + 3$$

$$= 4(x^2 + \Delta x^2 + 2x\Delta x) + 3$$

$$= 4x^2 + 4\Delta x^2 + 8x\Delta x + 3$$

$$\frac{\Delta y}{\Delta x} = \frac{4x^2 + 4\Delta x^2 + 8x\Delta x + 3 - (4x^2 + 3)}{\Delta x}$$

$$= \frac{\cancel{4x^2} + 4\Delta x^2 + 8x\Delta x \cancel{+3} - \cancel{4x^2} - \cancel{3}}{\Delta x}$$

$$= \frac{4\Delta x^2 + 8x\Delta x}{\Delta x} = \frac{\cancel{\Delta x}(4\Delta x + 8x)}{\cancel{\Delta x}}$$

$$= 4\Delta x + 8x$$

$$= 4(2) + 8(1)$$

$$= 8 + 8$$

$$= 16$$

$= \frac{\cancel{\Delta x}(10\Delta x + 20x + 20)}{\cancel{\Delta x}}$

$= 10\Delta x + 20x + 20$

$= 10(2) + 20(1) + 20$

$= 20 + 20 + 20$

$= 60$

25) $y = f(x) = 5$

$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$f(x+\Delta x) = 5$

$\frac{\Delta y}{\Delta x} = \frac{5 - 5}{\Delta x} = 0$

Slope = 0 because the constant functions always have the zero slope.

26) $y = f(x) = -3x^2 + 8x + 10$

$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$f(x+\Delta x) = -3(x+\Delta x)^2 + 8(x+\Delta x) + 10$

$= -3(x^2 + \Delta x^2 + 2x\Delta x) + 8(x+\Delta x) + 10$

$= -3x^2 - 3\Delta x^2 - 6x\Delta x + 8x + 8\Delta x + 10$

$\frac{\Delta y}{\Delta x} = \frac{-3x^2 - 3\Delta x^2 - 6x\Delta x + 8x + 8\Delta x + 10 - (-3x^2 + 8x + 10)}{\Delta x}$

$= \frac{\cancel{-3x^2} - 3\Delta x^2 - 6x\Delta x + \cancel{8x} + 8\Delta x + \cancel{10} + \cancel{3x^2} - \cancel{8x} - \cancel{10}}{\Delta x}$

$= \frac{-3\Delta x^2 - 6x\Delta x + 8\Delta x}{\Delta x}$

$= \frac{\cancel{\Delta x}(-3\Delta x - 6x + 8)}{\cancel{\Delta x}}$

$= -3\Delta x - 6x + 8$

$= -3(2) - 6(1) + 8$

$= -6 - 6 + 8$

$\Delta y / \Delta x = -4$

29) $y = f(x) = \frac{x^2}{3} + 5x$

$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$f(x+\Delta x) = \frac{(x+\Delta x)^2}{3} + 5(x+\Delta x)$

$= \frac{x^2 + \Delta x^2 + 2x\Delta x}{3} + 5x + 5\Delta x$

$= \frac{x^2 + \Delta x^2 + 2x\Delta x + 15x + 15\Delta x}{3}$

$\frac{\Delta y}{\Delta x} = \frac{\frac{x^2 + \Delta x^2 + 2x\Delta x + 15x + 15\Delta x}{3} - \left(\frac{x^2}{3} + 5x\right)}{\Delta x}$

$\frac{\Delta y}{\Delta x} = \frac{\cancel{x^2} + \Delta x^2 + 2x\Delta x + \cancel{15x} + 15\Delta x - \cancel{x^2} - \cancel{15x}}{3} / \Delta x$

$= \frac{\Delta x^2 + 2x\Delta x + 15\Delta x}{3\Delta x}$

$= \frac{\cancel{\Delta x}(\Delta x + 2x + 15)}{3\cancel{\Delta x}}$

$= \frac{\Delta x + 2x + 15}{3}$

$= \frac{2 + 2(1) + 15}{3} = \frac{19}{3}$

$\frac{\Delta y}{\Delta x} = 6.33$

35) $Y = f(x) = 5/x$

$$\frac{\Delta Y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x+\Delta x) = 5/(x+\Delta x)$$

$$\frac{\Delta Y}{\Delta x} = \frac{\frac{5}{x+\Delta x} - \frac{5}{x}}{\Delta x}$$

$$\frac{\Delta Y}{\Delta x} = \frac{\frac{5x - 5(x+\Delta x)}{(x+\Delta x)(x)}}{\Delta x}$$

$$= \frac{\cancel{5x} - \cancel{5x} - 5\Delta x}{(x+\Delta x)(x)} \times \frac{1}{\Delta x}$$

$$= \frac{-5\cancel{\Delta x}}{(x+\Delta x)(x)} \times \frac{1}{\cancel{\Delta x}}$$

$$= \frac{-5}{x^2 + x\Delta x}$$

$$= \frac{-5}{(1)^2 + 1(2)}$$

$$= \frac{-5}{1+3}$$

$$= -5/3$$

34) $Y = f(x) = 1/x$

$$\frac{\Delta Y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x+\Delta x) = 1/(x+\Delta x)$$

$$\frac{\Delta Y}{\Delta x} = \frac{\frac{1}{(x+\Delta x)} - \frac{1}{(x)}}{\Delta x}$$

$$= \frac{\frac{x-(x+\Delta x)}{(x+\Delta x)(x)}}{\Delta x}$$

$$= \frac{\cancel{x} - \cancel{x} + \cancel{\Delta x}}{(x+\Delta x)(x)} \times \frac{1}{\cancel{\Delta x}}$$

$$= \frac{1}{x^2 + x\Delta x}$$

b) $$= \frac{1}{(1)^2 + 1(2)}$$

$$= \frac{1}{1+2}$$

$$= \frac{1}{3}$$