<u>LECTURE # 28</u>

Mean Deviation, Standard Deviation and Variance & Coefficient of variation

- Mean Deviation
- Standard Deviation and Variance
- Coefficient of variation

First, we will discuss it for the case of raw data, and then we will go on to the case of a frequency distribution. The first thing to note is that, whereas the range as well as the quartile deviation are two such measures of dispersion which are NOT based on all the values, the mean deviation and the standard deviation are two such measures of dispersion that involve each and every data-value in their computation.

You must have noted that the range was measuring the dispersion of the data-set around the mid-range, whereas the quartile deviation was measuring the dispersion of the data-set around the median.

How are we to decide upon the amount of dispersion round the arithmetic mean? It would seem reasonable to compute the DISTANCE of each observed value in the series from the arithmetic mean of the series.

Let us do this for a simple data-set shown below:
The Number of Fatalities in Motorway Accidents in one Week:

Day	Number of fatalities X
Sunday	**4**
Monday	**6**
Tuesday	**2**
Wednesday	**0**
Thursday	**3**
Friday	**5**
Saturday	**8**
Total	**28**

Let us do this for a simple data-set shown below:
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The arithmetic mean number of fatalities per day is

$$\overline{X} = \frac{\sum X}{n} = \frac{28}{7} = 4$$

In order to determine the distances of the data-values from the mean, we subtract our value of the arithmetic mean from each daily figure, and this gives us the deviations that occur in the third column of the table below:

Day	Number of fatalities X	$X - \overline{X}$
Sunday	**4**	**0**
Monday	**6**	**+ 2**
Tuesday	**2**	**– 2**
Wednesday	**0**	**– 4**
Thursday	**3**	**– 1**
Friday	**5**	**+ 1**
Saturday	**8**	**+ 4**
TOTAL	**28**	**0**

The deviations are negative when the daily figure is less than the mean (4 accidents) and positive when the figure is higher than the mean.
It does seem, however, that our efforts for computing the dispersion of this data set have been in vain, for we find that the total amount of dispersion obtained by summing the $(x - \overline{x})$ column comes out to be zero! In fact, this should be no surprise, for it is a basic property of the arithmetic mean that:The sum of the deviations of the values from the mean is zero.
The question arises:

How will we measure the dispersion that is actually present in our data-set?

Our problem might at first sight seem irresolvable, for by this criterion it appears that no series has any dispersion. Yet we know that this is absolutely incorrect, and we must think of some other way of handling this situation. Surely, we might look at the numerical difference between the mean and the daily fatality figures without considering whether these are positive or negative. Let us denote these absolute differences by 'modulus of d' or 'mod d'.
This is evident from the third column of the table below:

X	$X - \overline{X} = d$	$\lvert d \rvert$
4	0	0
6	2	2
2	–2	2
0	–4	4
3	–1	1
5	1	1
8	4	4
	Total	14

By ignoring the sign of the deviations we have achieved a non-zero sum in our second column. Averaging these absolute differences, we obtain a measure of dispersion known as the mean deviation.
In other words, the mean deviation is given by the formula:
MEAN DEVIATION:

$$M.D. = \frac{\sum |d_i|}{n}$$

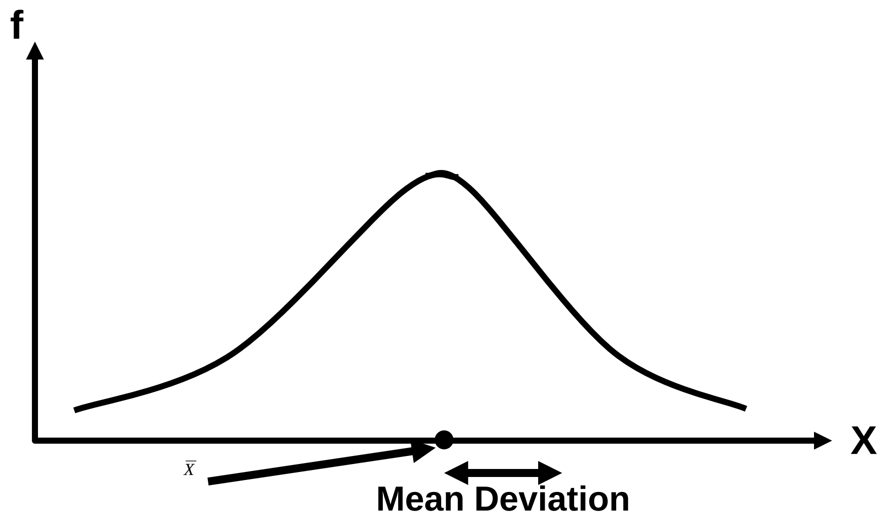
As we are averaging the absolute deviations of the observations from their mean, therefore the complete name of this measure is mean absolute deviation --- but generally we just say "mean deviation". Applying this formula in our example, we find that:
The mean deviation of the number of fatalities is

$$M.D. = \frac{14}{7} = 2.$$

The formula that we have just considered is valid in the case of raw data. In case of grouped data i.e. a frequency distribution, the formula becomes
MEAN DEVIATION FOR GROUPED DATA:

$$M.D. = \frac{\sum f_i \left| x_i - \bar{x} \right|}{n} = \frac{\sum f_i \left| d_i \right|}{n}$$

As far as the graphical representation of the mean deviation is concerned, it can be depicted by a horizontal line segment drawn below the X-axis on the graph of the frequency distribution, as shown below:



The approach which we have adopted in the concept of the mean deviation is both quick and simple. But the problem is that we introduce a kind of artificiality in its calculation by ignoring the algebraic signs of the deviations.

In problems involving descriptions and comparisons alone, the mean deviation can validly be applied; but because the negative signs have been discarded, further theoretical development or application of the concept is impossible.

Mean deviation is an absolute measure of dispersion. Its relative measure, known as the co-efficient of mean deviation, is obtained by dividing the mean deviation by the average used in the calculation of deviations i.e. the arithmetic mean. Thus

Co-efficient of M.D:

Sometimes, the mean deviation is computed by averaging the absolute deviations of the data-values from the median i.e.

$$= \frac{M.D.}{Mean}$$

$$\text{Mean deviation} = \frac{\sum |x - \tilde{x}|}{n}$$

And when will we have a situation when we will be using the median instead of the mean?As discussed earlier, the median will be more appropriate than the mean in those cases where our data-set contains a few very high or very low values.In such a situation, the coefficient of mean deviation is given by:

Co-efficient of M.D:

$$= \frac{\text{M.D.}}{\text{Median}}$$

Let us now consider the ***standard deviation*** --- that statistic which is the most important and the most widely used measure of dispersion.

The point that made earlier that from the mathematical point of view, it is not very preferable to take the absolute values of the deviations, *This problem is overcome by computing the standard deviation.*

In order to compute the standard deviation, rather than taking the absolute values of the deviations, we square the deviations.

Averaging these squared deviations, we obtain a statistic that is known as the variance.

Variance

$$= \frac{\sum \left(x - \overline{x}\right)^2}{n}$$

Let us compute this quantity for the data of the above example.
Our X-values were:

X
4
6
2
0
3
5
8

Taking the deviations of the X-values from their mean, and then squaring these deviations, we obtain:

X	$(x - \overline{x})$	$(x - \overline{x})^2$
4	0	0
6	+ 2	4
2	– 2	4
0	– 4	16
3	– 1	1
5	+ 1	1
8	+ 4	16
		42

Obviously, both (– 2)2 and (2)2 equal 4, both (– 4)2 and (4)2 equal 16, and both (– 1)2 and (1)2 = 1.

Hence $\sum(x - \bar{x})2 = 42$ is now positive, and this positive value has been achieved without 'bending' the rules of mathematics. Averaging these squared deviations, the variance is given by:
Variance:

$$= \frac{\sum (x - \bar{x})^2}{n}$$

$$= \frac{42}{7} = 6$$

The variance is frequently employed in statistical work, but it should be noted that the figure achieved is in 'squared' units of measurement.
In the example that we have just considered, the variance has come out to be "6 squared fatalities", which does not seem to make much sense!

In order to obtain an answer which is in the original unit of measurement, we take the positive square root of the variance. The result is known as the standard deviation.

STANDARD DEVIATION:

$$S = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

Hence, in this example, our standard deviation has come out to be 2.45 fatalities.

In computing the standard deviation (or variance) it can be tedious to first ascertain the arithmetic mean of a series, then subtract it from each value of the variable in the series, and finally to square each deviation and then sum.
It is very much more straight-forward to use the short cut formula given below:

SHORT CUT FORMULA FOR THE STANDARD DEVIATION:

$$S = \sqrt{\left\{ \frac{\sum x^2}{n} - \left(\frac{\sum x}{n} \right)^2 \right\}}$$

In order to apply the short cut formula, we require only the aggregate of the series ($\sum x$) and the aggregate of the squares of the individual values in the series ($\sum x2$).
In other words, only two columns of figures are called for. The number of individual calculations is also considerably reduced, as seen below:

	X	X^2
	4	16
	6	36
	2	4
	0	0
	3	9
	5	25
	8	64
Total	**28**	**154**

Therefore

$$S = \sqrt{\left\{\frac{154}{7} - \left(\frac{28}{7}\right)^2\right\}} = \sqrt{(22-16)}$$

$$= \sqrt{6} = 2.45 \text{ fatalities}$$

The formulae that we have just discussed are valid in case of raw data. In case of grouped data i.e. a frequency distribution, each squared deviation round the mean must be multiplied by the appropriate frequency figure i.e.

STANDARD DEVIATION IN CASE OF GROUPED DATA:

$$S = \sqrt{\frac{\sum f\left(x - \overline{x}\right)^2}{n}}$$

And the short cut formula in case of a frequency distribution is:
SHORT CUT FORMULA OF THE STANDARD DEVIATION IN CASE OF GROUPED DATA:

$$S = \sqrt{\left\{\frac{\sum fx^2}{n} - \left(\frac{\sum fx}{n}\right)^2\right\}}$$

Which is again preferred from the computational standpoint?
For example, the standard deviation life of a batch of electric light bulbs would be calculated as follows:
EXAMPLE:

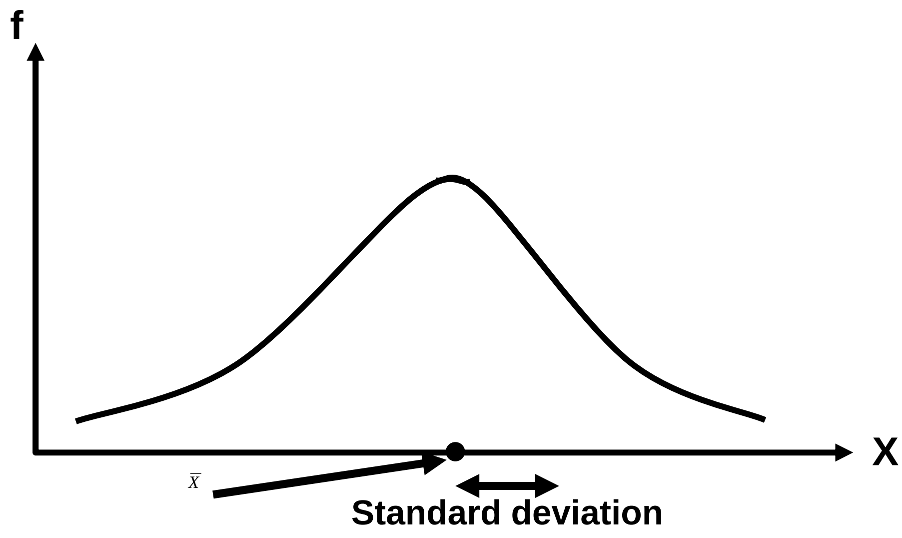
Life (in Hundreds of Hours)	No. of Bulbs f	Mid-point x	fx	fx^2
0 – 5	4	2.5	10.0	25.0
5 – 10	9	7.5	67.5	506.25
10 – 20	38	15.0	570.0	8550.0
20 – 40	33	30.0	990.0	29700.0
40 and over	16	50.0	800.0	40000.0

Therefore,
standard deviation:

$$S = \sqrt{\left\{\frac{78781.25}{100} - \left(\frac{2437.5}{100}\right)^2\right\}}$$

=13.9hundredhours
= 1390 hours

As far as the graphical representation of the standard deviation is concerned, a horizontal line segment is drawn below the X-axis on the graph of the frequency distribution --- just as in the case of the mean deviation.



The standard deviation is an absolute measure of dispersion. Its relative measure called coefficient of standard deviation is defined as:

Coefficient of S.D:

$$= \frac{\text{Standard Deviation}}{\text{Mean}}$$

And, multiplying this quantity by 100, we obtain a very important and well-known measure called the coefficient of variation.

Coefficient of Variation:

$$C.V. = \frac{S}{\overline{X}} \times 100$$

As mentioned earlier, the standard deviation is expressed in absolute terms and is given in the same unit of measurement as the variable itself.

There are occasions, however, when this absolute measure of dispersion is inadequate and a relative form becomes preferable.

For example, if a comparison between the variability of distributions with different variables is required, or when we need to compare the dispersion of distributions with the same variable but with very different arithmetic means.

To illustrate the usefulness of the coefficient of variation, let us consider the following two examples:

EXAMPLE-1

Suppose that, in a particular year, the mean weekly earnings of skilled factory workers in one particular country were $ 19.50 with a standard deviation of $ 4, while for its neighboring country the figures were Rs. 75 and Rs. 28 respectively.

From these figures, it is not immediately apparent which country has the GREATER VARIABILITY in earnings.

The coefficient of variation quickly provides the answer:

COEFFICIENT OF VARIATION

For country No. 1:

$$\frac{4}{19.5} \times 100 = 20.5 \text{ per cent,}$$

And for country No. 2:

$$\frac{28}{75} \times 100 = 37.3 \text{ per cent.}$$

From these calculations, it is immediately obvious that the spread of earnings in country No. 2 is greater than that in country No. 1, and the reasons for this could then be sought.

EXAMPLE-2:

The crop yield from 20 acre plots of wheat-land cultivated by ordinary methods averages 35 bushels with a standard deviation of 10 bushels. The yield from similar land treated with a new fertilizer averages 58 bushels, also with a standard deviation of 10 bushels. At first glance, the yield variability may seem to be the same, but in fact it has improved (i.e. decreased) in view of the higher average to which it relates.

Again, the coefficient of variation shows this very clearly:

Coefficient of Variation:

Untreated land:

$$\frac{10}{35} \times 100 = 28.57 \text{ per cent}$$

Treated land:

$$\frac{10}{58} \times 100 = 17.24 \text{ per cent}$$

The coefficient of variation for the untreated land has come out to be 28.57 percent, whereas the coefficient of variation for the treated land is only 17.24 percent.